D INFERENCE FOR CATEGORICAL DATA

ıesis

:ondary Infection

No	Total
$\pi(1-\pi)$	π
$1-\pi$	$1-\pi$

$)^{n_{12}}(1-\pi)^{n_{22}}.$

$\pi-\pi^2)+n_{22}\log(1-\pi).$

; the likelihood equation

$\frac{\cdot 12}{-\pi} - \frac{n_{22}}{1-\pi} = 0.$

$(2n_{11}+2n_{12}+n_{22}).$

$= 156$, the estimated expected frequen- $\hat{\pi}-\hat{\pi}^2) = 39.0$, and $\hat{\mu}_{22} = n(1-\hat{\pi}) =$'s statistic is $X^2 = 19.7$. Since the $c = 3$ ameter (π) determining the expected here is strong evidence against H_0 ($P =$ ɛals that many more calves got a primary tion than H_0 predicts. The researchers ı had an immunizing effect that reduced on.

ıal, interval) scales of measurement. Other scales For instance, *partially ordered* scales occur when tegories ordered except for don't know or undecided

:al Data

ıus, when β is a model parameter, one can usually ıg H_0: $\beta = \beta_0$ without fitting the model. This is ›dels in an exploratory analysis and model fitting is ıge of the score and likelihood-ratio methods is that

they apply even when $|\hat{\beta}| = \infty$. In that case, one cannot compute the Wald statistic. Another disadvantage of the Wald method is that its results depend on the parameterization; inference based on $\hat{\beta}$ and its SE is not equivalent to inference based on a nonlinear function of it, such as log $\hat{\beta}$ and its SE.

Section 1.4: Statistical Inference for Binomial Parameters

1.3. Among others, Agresti and Coull (1998), Blyth and Still (1983), Brown et al. (2001), Ghosh (1979), and Newcombe (1998a) showed the superiority of the score interval to the Wald interval for π. Of the "exact" methods, Blaker's (2000) has particularly good properties. It is contained in the Clopper–Pearson interval and has a nestedness property whereby an interval of higher nominal confidence level necessarily contains one of lower level.

1.4. Using continuity corrections with large-sample methods provides approximations to exact small-sample methods. Thus, they tend to behave conservatively. We do not present them, since if one prefers an exact method, with modern computational power it can be used directly rather than approximated.

1.5. In theory, one can eliminate problems with discreteness in tests by performing a supplementary randomization on the boundary of a critical region (see Problem 1.19). In rejecting the null at the boundary with a certain probability, one can obtain a fixed overall type I error probability α even when it is not an achievable P-value. For such randomization, the one-sided P − value is

$$\text{randomized } P\text{-value} = U \times P(T = t_o) + P(T > t_o),$$

where U denotes a uniform (0, 1) random variable (Stevens 1950). In practice, this is not used, as it is absurd to let this random number influence a decision. The mid P-value replaces the arbitrary uniform multiple $U \times P(T = t_o)$ by its expected value.

Section 1.5: Statistical Inference for Multinomial Parameters

1.6. The chi-squared distribution has mean df, variance 2 df, and skewness $(8/\text{df})^{1/2}$. It is approximately normal when df is large. Greenwood and Nikulin (1996), Kendall and Stuart (1979), and Lancaster (1969) presented other properties. Cochran (1952) presented a historical survey of chi-squared tests of fit. See also Cressie and Read (1989), Koch and Bhapkar (1982), Koehler (1998), and Moore (1986b).

PROBLEMS

Applications

1.1 Identify each variable as nominal, ordinal, or interval.

a. UK political party preference (Labour, Conservative, Social Democrat)

b. Anxiety rating (none, mild, moderate, severe, very severe)

c. Patient survival (in number of months)

d. Clinic location (London, Boston, Madison, Rochester, Montreal)

e. Response of tumor to chemotherapy (complete elimination, partial reduction, stable, growth progression)

f. Favorite beverage (water, juice, milk, soft drink, beer, wine)

g. Appraisal of company's inventory level (too low, about right, too high)

1.2 Each of 100 multiple-choice questions on an exam has four possible answers, one of which is correct. For each question, a student guesses by selecting an answer randomly.

a. Specify the distribution of the student's number of correct answers.

b. Find the mean and standard deviation of that distribution. Would it be surprising if the student made at least 50 correct responses? Why?

c. Specify the distribution of (n_1, n_2, n_3, n_4), where n_j is the number of times the student picked choice j.

d. Find $E(n_j)$, $\text{var}(n_j)$, $\text{cov}(n_j, n_k)$, and $\text{corr}(n_j, n_k)$.

1.3 An experiment studies the number of insects that survive a certain dose of an insecticide, using several batches of insects of size n each. The insects are sensitive to factors that vary among batches during the experiment but were not measured, such as temperature level. Explain why the distribution of the number of insects per batch surviving the experiment might show overdispersion relative to a $\text{bin}(n, \pi)$ distribution.

1.4 In his autobiography *A Sort of Life*, British author Graham Greene described a period of severe mental depression during which he played Russian Roulette. This "game" consists of putting a bullet in one of the six chambers of a pistol, spinning the chambers to select one at random, and then firing the pistol once at one's head.

a. Greene played this game six times and was lucky that none of them resulted in a bullet firing. Find the probability of this outcome.

b. Suppose that he had kept playing this game until the bullet fired. Let Y denote the number of the game on which it fires. Show the probability mass function for Y, and justify.

1.5 Consider the statement, "Please tell me whether or not you think it should be possible for a pregnant woman to obtain a legal abortion if she is married and does not want any more children." For the 1996 General Social Survey, conducted by the National Opinion Research Center (NORC), 842 replied "yes" and 982 replied "no." Let π denote

TABLE 1.3 Data for Problem 1.9

Number of Deaths	Number of Corps-Years
0	109
1	65
2	22
3	3
4	1
≥ 5	0

1.10 A sample of 100 women suffer from dysmenorrhea. A new analgesic is claimed to provide greater relief than a standard one. After using each analgesic in a crossover experiment, 40 reported greater relief with the standard analgesic and 60 reported greater relief with the new one. Analyze these data.

Theory and Methods

1.11 Why is it easier to get a precise estimate of the binomial parameter π when it is near 0 or 1 than when it is near $\frac{1}{2}$?

1.12 Suppose that $P(Y_i = 1) = 1 - P(Y_i = 0) = \pi$, $i = 1, \ldots, n$, where $\{Y_i\}$ are independent. Let $Y = \sum_i Y_i$.

a. What are $\text{var}(Y)$ and the distribution of Y?

b. When $\{Y_i\}$ instead have pairwise correlation $\rho > 0$, show that $\text{var}(Y) > n\pi(1 - \pi)$, overdispersion relative to the binomial. [Altham (1978) discussed generalizations of the binomial that allow correlated trials.]

c. Suppose that heterogeneity exists: $P(Y_i = 1|\pi) = \pi$ for all i, but π is a random variable with density function $g(\cdot)$ on [0, 1] having mean ρ and positive variance. Show that $\text{var}(Y) > n\rho(1 - \rho)$. (When π has a beta distribution, Y has the *beta-binomial distribution* of Section 13.3.)

d. Suppose that $P(Y_i = 1|\pi_i) = \pi_i$, $i = 1, \ldots, n$, where $\{\pi_i\}$ are independent from $g(\cdot)$. Explain why Y has a $\text{bin}(n, \rho)$ distribution unconditionally but not conditionally on $\{\pi_i\}$. (*Hint:* In each case, is Y a sum of independent, identical Bernoulli trials?)

1.13 For a sequence of independent Bernoulli trials, Y is the number of successes before the kth failure. Explain why its probability mass