1.9

Number of Corps-Years
109
65
22
3
1
0

om dysmenorrhea. A new analgesic is than a standard one. After using each nt, 40 reported greater relief with the rted greater relief with the new one.

estimate of the binomial parameter π n it is near $\frac{1}{2}$?

$P(Y_i = 0) = \pi$, $i = 1, \dots, n$, where $\{Y_i\}$

stribution of Y?

airwise correlation $\rho > 0$, show that lispersion relative to the binomial. [Al- eralizations of the binomial that allow

exists: $P(Y_i = 1|\pi) = \pi$ for all i, but π ensity function $g(\cdot)$ on [0, 1] having mean ow that $\text{var}(Y) > n\rho(1-\rho)$. (When π has the *beta-binomial distribution* of

$= \pi_i$, $i = 1, \dots, n$, where $\{\pi_i\}$ are inde- in why Y has a bin(n, ρ) distribution nditionally on $\{\pi_i\}$. (*Hint:* In each case, is dentical Bernoulli trials?)

ent Bernoulli trials, Y is the number of ilure. Explain why its probability mass

function is the *negative binomial*,

$$p(y) = \frac{(y+k-1)!}{y!(k-1)!}\pi^y(1-\pi)^k, \qquad y = 0, 1, 2, \dots .$$

[For it, $E(Y) = k\pi/(1-\pi)$ and $\text{var}(Y) = k\pi/(1-\pi)^2$, so $\text{var}(Y) > E(Y)$; the Poisson is the limit as $k \to \infty$ and $\pi \to 0$ with $k\pi = \mu$ fixed.]

1.14 For the multinomial distribution, show that

$$\text{corr}(n_j, n_k) = -\pi_j\pi_k/\sqrt{\pi_j(1-\pi_j)\pi_k(1-\pi_k)}\,.$$

Show that $\text{corr}(n_1, n_2) = -1$ when $c = 2$.

1.15 Show that the moment generating function (mgf) for the binomial distribution is $m(t) = (1 - \pi + \pi e^t)^n$, and use it to obtain the first two moments. Show that the mgf for the Poisson distribution is $m(t) = \exp(\mu[\exp(t) - 1]\}$, and use it to obtain the first two moments.

1.16 A likelihood-ratio statistic equals t_o. At the ML estimates, show that the data are $\exp(t_o/2)$ times more likely under H_a than under H_0.

1.17 Assume that $y_1, y_2, \dots, y_n$ are independent from a Poisson distribution.

a. Obtain the likelihood function. Show that the ML estimator $\hat{\mu} = \bar{y}$.

b. Construct a large-sample test statistic for H_0: $\mu = \mu_0$ using **(i)** the Wald method, **(ii)** the score method, and **(iii)** the likelihood-ratio method.

c. Construct a large-sample confidence interval for μ using **(i)** the Wald method, **(ii)** the score method, and **(iii)** the likelihood-ratio method.

1.18 Inference for Poisson parameters can often be based on connections with binomial and multinomial distributions. Show how to test H_0: $\mu_1 = \mu_2$ for two populations based on independent Poisson counts (y_1, y_2), using a corresponding test about a binomial parameter π. [*Hint:* Condition on $n = y_1 + y_2$ and identify $\pi = \mu_1/(\mu_1 + \mu_2)$.] How can one construct a confidence interval for μ_1/μ_2 based on one for π?

1.19 A researcher routinely tests using a nominal P(type I error) = 0.05, rejecting H_0 if the P-value ≤ 0.05. An exact test using test statistic T