c. Use the pivotal method for deriving confidence intervals to show that the roots in π of

$$\frac{\alpha}{2} = \sum_{x=0}^{S} \binom{n}{x} \pi^x (1-\pi)^{n-x} \quad \frac{\alpha}{2} = \sum_{x=S}^{n} \binom{n}{x} \pi^x (1-\pi)^{n-x}$$

determine an interval having confidence coefficient at least $1-\alpha$.

3.30 When data have the multinomial $(n, \{\pi_i, i = 1, \ldots, N\})$ distribution, with $N > 2$, we can obtain confidence limits for π_i as the solutions of

$$(p_i - \pi_i)^2 = (z_{\alpha/2N})^2 \pi_i (1-\pi_i)/n, \; i = 1, \ldots, N.$$

a. Using the Bonferroni inequality, argue that these N confidence intervals hold simultaneously (for large samples) with probability at least $1-\alpha$.

b. Show that the standard deviation of $p_i - p_j$ is $[\pi_i + \pi_j - (\pi_i - \pi_j)^2]/n$. For large n, show that the probability is at least $1-\alpha$ that the confidence intervals

$$(p_i - p_j) \pm z_{\alpha/2a} \{[p_i + p_j - (p_i - p_j)^2]/n\}^{1/2}$$

simultaneously contain the $a = N(N-1)/2$ differences $\{\pi_i - \pi_j\}$. See Goodman (1965), Fitzpatrick and Scott (1987), and Miller (1981).

3.31 Assuming multinomial sampling, show that $\sigma^2 = (\Sigma\Sigma\, \pi_{ij}^{-1})(1-Q^2)^2/4$ in (3.21) for Yule's Q. This result was noted by Yule (1900, 1912).

3.32 Suppose a measure ζ defined for a two-way table has form $\zeta = \nu/\delta$. For multinomial sampling, show the asymptotic variance of $\sqrt{n}(\hat{\zeta} - \zeta)$ is $\sigma^2 = [\Sigma\Sigma\, \pi_{ij}\eta_{ij}^2 - (\Sigma\Sigma\, \pi_{ij}\eta_{ij})^2]/\delta^4$, where $\eta_{ij} = \delta(\partial\nu/\partial\pi_{ij}) - \nu(\partial\delta/\partial\pi_{ij})$ (Goodman and Kruskal, 1972).

3.33 Refer to Note 3.6. For independent multinomial sampling with log θ, show that ϕ_{ij}^{+} equals $1/\pi_{j|i}$ when $i = j$ and $-1/\pi_{j|i}$ when $i \neq j$. Conclude that $\sigma^2 = \Sigma\Sigma\, 1/\omega_i \pi_{j|i}$, the same as the asymptotic variance for multinomial sampling when $\omega_i = \pi_{i+}$.

3.34 Explain why $\{n_{+j}\}$ are sufficient for $\{\pi_{+j}\}$ in (3.24).