Stat 545 Part II Homework # 1 Solution

Solution to Problem # 1.

```
proc logistic data = basketball desc ;
model win = n_assist / lackfit ;
output out = predict pred = prob ;
run ;

proc sort data = predict ; by n_assist win ; run ;

proc print data = predict ;
run ;
```

Solution to Problem # 2.

$$\left|\frac{\partial \pi(x)}{\partial x}\right| = |\beta|\pi(x)[1-\pi(x)] \leq \frac{|\beta|}{2}$$

The equality holds when $\pi(x) = 1/2$.

Solution to Problem # 3.

The log likelihood is

$$L(\alpha, \beta) = \sum_{i=0}^{1} y_i(\alpha + \beta x_i) - \sum_{i=0}^{1} n_i \log\left[1 + \exp(\alpha + \beta x_i)\right]$$

where $x_0 = 0$, $x_1 = 1$.

The likelihood equation is:

$$\frac{\partial L(\alpha, \beta)}{\partial \alpha} = y_0 - n_0 \frac{\exp(\alpha)}{1 + \exp(\alpha)} = 0$$
$$\frac{\partial L(\alpha, \beta)}{\partial \beta} = y_1 - n_1 \frac{\exp(\alpha + \beta)}{1 + \exp(\alpha + \beta)} = 0$$

Solving these equations, we have

$$\exp(\alpha) = \frac{y_0/n_0}{(n_0 - y_0)/n_0}$$
$$\exp(\alpha + \beta) = \frac{y_1/n_1}{(n_1 - y_1)/n_1}$$

and hence,

$$\exp(\beta) = \frac{y_1/n_1}{(n_1 - y_1)/n_1} \bigg/ \frac{y_0/n_0}{(n_0 - y_0)/n_0}$$

Solution to Problem # 4.

Let $x_{ij} = 1$ and $n_i = 1$, Equation (5.18) becomes

$$\sum_i y_i - \sum_i \hat{\pi}_i = 0$$

which implies

$$n^{-1} \sum_i y_i = n^{-1} \sum_i \hat{\pi}_i$$

Solution to Problem # 5.

(a) The log odds ratio of a covariate is $\log(\hat{\theta})$ with 95% confidence interval $(\log(\hat{\theta}_L), \log(\hat{\theta}_U))$. Let $\hat{\sigma}$ denote the estimated standard error. $\log(\hat{\theta}_L) = \log(\hat{\theta}) - 1.96\hat{\sigma}$, $\log(\hat{\theta}_U) = \log(\hat{\theta}) + 1.96\hat{\sigma}$.

$$\hat{\sigma} = \frac{\log(\hat{\theta}_U - \log(\hat{\theta}_L)}{2 \times 1.96}$$

The standard errors of Group, Gender, and SES are 0.6313482, 0.5991484, 0.5815993.

(b) For gender, the log odds ratio is $\log(1.38) = 0.3220835$. The 95% CI of log odds ratio is $(\log(1.23), \log(12.88)) = (0.2070142, 2.5556757)$. The average of 0.2070142 and 2.5556757 is 1.381345. If the reported CI is correct, the log odds ratio should be 1.381345 and the odds ratio is 3.980251.